

From editor

This issue is two years late. We hope the next issue will be sooner.

We shall clear what is the relational logic.

The relational logic replaces functions and predicates by relations. In result the extra distinctions between functions and predicates are erased. In particular, predicates may be terms.

In this case the relational logic is an expansion of the classic logic, and the classical logic in all its variety is a part of the relational logic.

But there is also a natural relational logic. This logic eliminates all unnatural in classic logic.

The classic set theory is elegant, but unnatural. More precisely, it is not a theory, it is a metatheory, since all mathematical theories are syntactic constructions, and the set theory is a semantic construction. Nevertheless, we shall follow to existing traditions.

The most natural set theory is the next one. All sets are constructed from the set of natural numbers \mathbb{N} , and with a finite number of operations of the direct product and power set. Subsets of these sets are the sets, too. Other sets do not exist. In particular, natural numbers and any tuples are not the sets. All these objects are “empty sets”, therefore all “empty sets” are not the sets. Nevertheless, it is possible to use the symbol of empty set \emptyset . For example, $R_1 \cap R_2 = \emptyset$ is a brief record $\forall R_0 R_1 \cap R_2 \neq R_0$, where R_1, R_2, R_0 are variables, and values of these variables are names of the sets.

The natural set theory is rather powerful theory, in spite of simplicity. There exists an arithmetic universality, and it is not a set, but containing all sets as members. There is algebraic universality, and it contains all arithmetic subuniversalities. It is possible to construct universalities any power if this is necessary.

The natural set theory, together with universalities, does not contain paradoxes.

The classical set theory contains paradoxes, yet not found out. In particular, there are subsets of \mathbb{N} such that members of these subsets can not be numbered. These are productive and immune sets. Hence, there are powers, intermediate between the finite and countable powers. But ordinals show that such intermediate powers are not exist. Only attraction of the choice axiom allows to resolve this paradox.

The natural set theory allows to get results, and these results can not be get in the classical theory. An example is countable ordinals.

A countable ordinal is a family of the well-ordered sets of rational numbers with the same order type. Rational numbers are members of \mathbb{N}^3 since the numerator is an integer, i.e., a member of \mathbb{N}^2 , the denominator is a positive natural number, i.e., a member of \mathbb{N} . The well-ordered set of rational numbers is a member of $\mathcal{P}(\mathbb{N}^3)$. Hence, a countable ordinal is a member of $\mathcal{P}^2(\mathbb{N}^3)$. The set of all countable ordinals is a member of $\mathcal{P}^3(\mathbb{N}^3)$. There is no base to believe that this member is uncountable. Moreover, in one of future articles it will be shown that this member is countable. But it does not mean, that the classic set theory is inconsistent. It means only that the classic theory is a fiction because this theory does not fit to our conceptions of sets.

One more elegant, but unnatural, theory is the non-standard analysis. The most natural theory of the non-standard analysis are based on the Cantor model. The corresponding article opens this issue.

Let's note the Hilbert space exists in the natural set theory in spite of a finite number of direct products. Instead of direct products we use an infinite sequence of real numbers as a point in the Hilbert space. A real number in this sequence is a member of $\mathcal{P}(\mathbb{N}^3)$. An element of the sequence is a member of $\mathcal{P}(\mathbb{N}^3) \times \mathbb{N}$. So the sequence is a member of $\mathcal{P}(\mathcal{P}(\mathbb{N}^3) \times \mathbb{N})$, and the Hilbert space is a member of $\mathcal{P}^2(\mathcal{P}(\mathbb{N}^3) \times \mathbb{N})$.